Calculer les intégrales suivantes (Tous les coups sont permis!)

$$I_1 = \int_0^3 (2t^2 - t + 4)\,dt = \left[\frac{2t^3}{3} - \frac{t^2}{2} + 4t\right]_0^3 = \left(\frac{2\times 3^3}{3} - \frac{3^2}{2} + 4\times 3\right) - 0 = \frac{4\times 9 - 9 + 24}{2} = \frac{51}{2}$$

$$I_2 = \int_{\frac{3}{2}}^5 \frac{1}{2t-1}\,dt = \left[\frac{\ln(2t-1)}{2}\right]_{\frac{3}{2}}^5 = \frac{\ln(9)}{2} - \frac{\ln 2}{2} = \ln(3) - \frac{\ln(2)}{2}$$

$$I_3 = \int_0^{\pi/2} x\sin(2x)\,dx$$

on intègre par parties : $u = x$, $u' = 1$, $v' = \sin(2x)$, $v = -\frac{\cos(2x)}{2}$

$$= \left[-\frac{x\cos(2x)}{2}\right]_0^{\pi/2} - \int_0^{\pi/2} -\frac{\cos(2x)}{2}\,dx = \frac{\pi}{4} + 0 + \frac{1}{2}\left[\frac{\sin(2x)}{2}\right]_0^{\pi/2} = \frac{\pi}{4} + 0 = \frac{\pi}{4}$$

$$I_4 = \int_2^5 \left(3x + 4 + \frac{1}{x}\right)dx = \left[3\frac{x^2}{2} + 4x + \ln|x|\right]_2^5 = \left(\frac{75}{2} + 20 + \ln 5\right) - (6 + 8 + \ln 2)$$
$$= \frac{87}{2} + \ln(5) - \ln(2)$$

$$I_5 = \int_0^1 e^{2t}\,dt = \left[\frac{e^{2t}}{2}\right]_0^1 = \frac{e^2}{2} - \frac{1}{2} = \frac{e^2 - 1}{2}$$

$$I_6 = \int_0^1 t e^{2t}\,dt$$

on intègre par parties : $u = t$, $u' = 1$, $v' = e^{2t}$, $v = \frac{e^{2t}}{2}$

$$= \left[\frac{te^{2t}}{2}\right]_0^1 - \int_0^1 \frac{e^{2t}}{2}\,dt = \frac{e^2}{2} - 0 - \left[\frac{e^{2t}}{4}\right]_0^1 = \frac{e^2}{2} - \left(\frac{e^2}{4} - 1\right) = 1 - \frac{e^2}{4}$$

$$I_7 = \int_1^2 \frac{e^x}{e^x - 1}\,dx = \left[\ln|e^x - 1|\right]_1^2 = \ln(e^2 - 1) - \ln(e-1) = \ln\left(\frac{e^2-1}{e-1}\right) = \ln\left(\frac{(e-1)(e+1)}{e-1}\right)$$
$$= \ln(e+1)$$

$$I_8 = \int_{\pi/4}^{\pi/2} \sin(3t)\,dt = \left[-\frac{\cos(3t)}{3}\right]_{\pi/4}^{\pi/2} = -\frac{\cos\left(\frac{3\pi}{2}\right)}{3} + \frac{\cos\left(\frac{3\pi}{4}\right)}{3} = 0 - \frac{\sqrt{2}}{6} = -\frac{\sqrt{2}}{6}$$

$$I_9 = \int_1^e (x - e)\ln(x)\,dx$$

on intègre par parties : $v' = x - e$, $v = x^2 - ex$, $u = \ln(x)$, $u' = \frac{1}{x}$

$$= \left[(x^2 - ex)\ln x\right]_1^e - \int_1^e (x - e)\,dx = 0 - 0 - \left[\frac{x^2}{2} - ex\right]_1^e$$
$$= -\left(\left(\frac{e^2}{2} - e^2\right) - \left(\frac{1}{2} - e\right)\right) = \frac{e^2}{2} - e + \frac{1}{2}$$

$$I_{10} = \int_0^2 (1-x)e^{-x}\,dx$$

on intègre par parties : $u = (1-x)$, $u' = -1$, $v' = e^{-x}$, $v = -e^{-x}$

$$I_{10} = \left[(x-1)e^{-x}\right]_0^2 - \int_0^2 e^{-x}\,dx = e^{-2} + 1 - \left[-e^{-x}\right]_0^2 = e^{-2} + 1 + e^{-2} - 1 = 2e^{-2}$$